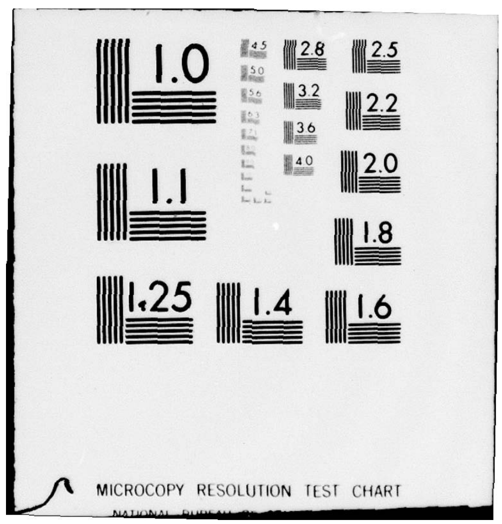
AD-A074 637 STANFORD UNIV CA DIV OF APPLIED MECHANICS F/G 13/11
ON HARMONIC MOTION OF SANDWICH PLATES.(U)
JUN 79 G HERRMANN, G S BEAUPRE N00014-76-C-0054
UNCLASSIFIED SUDAM-79-2 NL

| OF |
ADA
074637

END
DATE
FILMED
10-79
DDC


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
MICROCOPY RESOLUTION TEST CHART

Office of Naval Research
Department of the Navy
Scientific Report
Contract N00014-76-C-0054

ON HARMONIC MOTION OF SANDWICH PLATES

by

G. Herrmann

G. S. Beaupre

SUDAM Report No. 79-2

June 1979

DIVISION
OF
APPLIED
MECHANICS

DEPARTMENT
OF
MECHANICAL
ENGINEERING

Die Luft der Freiheit weht

STANFORD
UNIVERSITY
STANFORD,
CALIFORNIA
94305

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER (14) SUDAM-79-2	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle) (6) ON HARMONIC MOTION OF SANDWICH PLATES.	5. TYPE OF REPORT & PERIOD COVERED (9) INTERIM rept.
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) (10) G. HERRMANN, G. S. BEAUPRE	8. CONTRACT OR GRANT NUMBER(s) (15) N00014-76-C-0054
9. PERFORMING ORGANIZATION NAME AND ADDRESS DIVISION OF APPLIED MECHANICS, STANFORD UNIVERSITY, STANFORD, CALIFORNIA 94305	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS ONR:474
11. CONTROLLING OFFICE NAME AND ADDRESS OFFICE OF NAVAL RESEARCH, DEPARTMENT OF THE NAVY, ARLINGTON, VIRGINIA 22217	12. REPORT DATE (11) JUNE 1979
	13. NUMBER OF PAGES 21
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) (12) 21	15. SECURITY CLASS. (of this report) UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

COMPOSITES
LAMINATES
SANDWICH PLATES
WAVE PROPAGATION

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Horizontally polarized shear motion in symmetric sandwich plates is considered. It is shown that certain Floquet-type solutions describing such motion in an infinite, periodically layered, elastic composite describe the same motion.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED 410 389

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

ON HARMONIC MOTION OF SANDWICH PLATES

Abstract

Horizontally polarized shear motion in symmetric sandwich plates is considered. It is shown that certain Floquet-type solutions describing such motion in an infinite, periodically layered, elastic composite describe the same motion.

Introduction

Dispersion relations for geometrically symmetric sandwich plates undergoing SH wave motion are obtained in two ways. First the equations of elasticity are used to formulate the corresponding boundary value problem for each layer which is then solved to obtain the dispersion relation. Second the same dispersion relation is obtained by inspection of the edges of the Brillouin zones for the problem involving SH wave propagation in an infinite medium consisting of two alternating layers made of the same material as for the sandwich plate.

Boundary Value Problem Formulation

A symmetric sandwich plate consists of two facings of the same thickness and material bonded to each side of a core made of a different material (see fig. 1). Both the core and the facings are assumed to be isotropic and linearly elastic. Since we will be considering only the propagation of SH waves within the sandwich plates, the specification of three geometric and material parameters is sufficient for their mathematical description. These parameters are: $\varepsilon = h'/h$, the ratio of half the thickness of the core to the thickness of one of the faces, $\gamma = \mu'/\mu$, the ratio of shear moduli, and $\sigma^2 = \mu\rho'/\mu'\rho$, the square of the ratio of shear wave speeds. The primed quantities refer to properties of the core and the unprimed quantities refer to properties of the faces.

In this analysis we will examine symmetric sandwich plates constrained by symmetric boundary conditions. In this case only symmetric and antisymmetric motions with respect to the sandwich plate midplane are possible. Four distinct boundary value problems will now be posed. The first two will deal with sandwich plates having stress-free faces considering both symmetric and antisymmetric motion with respect to the core midplane. The second two involve displacement-free [clamped] faces, again considering both symmetric and antisymmetric motions. In each case six boundary conditions [two boundary conditions on the faces and four interface conditions] are necessary to completely specify the problem. By taking into account symmetry [or antisymmetry] of the problem this number may be reduced to four.

As an example consider the problem in which the faces are stress-free and the motion is antisymmetric with respect to the core midplane. Let u,v,w be the displacement components in the x,y,z directions respectively.

For SH wave motion we take v=w=o and u = u(y,z,t). The differential equation which governs the motion in the top face is

$$\mu \left(\frac{d^2u}{dy_1^2} + \frac{d^2u}{dz^2}\right) = \rho\ddot{u} \qquad -h \le y_1 \le 0 \tag{1}$$

where y_1 is measured from the upper edge of the top face (see Fig. 1). For plane waves travelling in the positive z direction we assume

$$u = f(y_1)e^{i(k_z z-\omega t)} \tag{2}$$

where k_z is the wave number in the z direction and ω is the frequency in radians per unit of time. Substitution of (2) into (1) yields the ordinary differential equation

$$\frac{d^2f}{dy_1^2} + \left(\frac{\rho\omega^2}{\mu} - k_z^2\right) f = 0 \tag{3}$$

the solution for which may be written

$$f(y_1) = C_1 \exp\left(\frac{i\Pi\alpha y_1}{2h}\right) + C_1' \exp\left(-\frac{i\Pi\alpha y_1}{2h}\right) \tag{4}$$

Here $\alpha = \sqrt{\Omega^2 - \zeta^2}$ where Ω and ζ are, respectively, the nondimensional frequency and wave number defined by

$$\Omega = \frac{2h\omega}{\Pi\sqrt{\frac{\mu}{\rho}}} \; ; \; \zeta = \frac{2h}{\Pi} k_z \tag{5}$$

The displacement in the top face now takes the form

$$u_1(y_1\ z,\ t) = \left[C_1 \exp\left(\frac{i\Pi\alpha y_1}{2h}\right) + C_1' \exp\left(\frac{-i\Pi\alpha y_1}{2h}\right)\right] \times \exp\left[\frac{i\Pi}{2}\left(\frac{\zeta z}{h} - \sqrt{\frac{\mu}{\rho}}\frac{\Omega t}{h}\right)\right] \tag{6}$$

In a similar fashion the displacement in the core may be written as

$$u_2\ (y_2\ z,\ t) = \left[C_2 \exp\left(\frac{i\Pi\alpha' y_2}{2h}\right) + C_2' \exp\left(\frac{-i\Pi\alpha' y_2}{2h}\right)\right] \times \exp\left[\frac{i\Pi}{2}\left(\frac{\zeta z}{h} - \sqrt{\frac{\mu}{\rho}}\ \frac{\Omega t}{h}\right)\right] \tag{7}$$

where $\alpha' = \sqrt{\sigma^2\Omega^2-\zeta^2}$ with $\sigma^2 = \mu\rho'/\mu'\rho$ and where $-h' \leq y_2 \leq h'$.

The displacement in the lower face is similar to equation (6) with C_1, C_1' replaced by C_3, C_3' and y_1 replaced by y_3 where $0 \leq y_3 \leq h$.

Using these displacement equations along with the isotropic stress strain relations $\sigma_{xy} = \mu\left(\frac{\partial u}{\partial y}\right)$ we may write expressions for the stress and displacement at any location within the plate.

Let us now consider the boundary value problem for a symmetric sandwich plate whose faces are traction-free in which the motion is antisymmetric with respect to the core midplane. The boundary conditions consist of the stress being zero at the outer edge of each facing and that

the stress and displacement are continuous at each core-facing interface. These conditions are described by the following six equations.

$$
\begin{aligned}
\sigma_{xy_1}\Big|_{y_1 = 0} &= 0 \\
u_1\Big|_{y_1 = -h} &= u_2\Big|_{y_2 = h'} \\
\sigma_{xy_1}\Big|_{y_1 = -h} &= \sigma_{xy_2}\Big|_{y_2 = h'} \\
u_2\Big|_{y_2 = -h'} &= u_3\Big|_{y_3 = h} \\
\sigma_{xy_2}\Big|_{y_2 = -h'} &= \sigma_{xy_3}\Big|_{y_3 = h} \\
\sigma_{xy_3}\Big|_{y_3 = 0} &= 0
\end{aligned}
\tag{8}
$$

These six boundary conditions lead to six homogeneous equations. For a nontrivial solution the determinant of the coefficients of the matrix formed by these equations must be zero. The determinant takes the following form.

$$0 = \begin{vmatrix} \alpha & -\alpha & 0 & 0 & 0 & 0 \\ e^{\frac{-i\Pi\alpha}{2}} & e^{\frac{i\Pi\alpha}{2}} & -e^{\frac{i\Pi\alpha'\varepsilon}{2}} & -e^{\frac{-i\Pi\alpha'\varepsilon}{2}} & 0 & 0 \\ \mu\alpha e^{\frac{-i\Pi\alpha}{2}} & -\mu\alpha e^{\frac{i\Pi\alpha}{2}} & -\mu'\alpha' e^{\frac{i\Pi\alpha'\varepsilon}{2}} & \mu'\alpha' e^{\frac{-i\Pi\alpha'\varepsilon}{2}} & 0 & 0 \\ 0 & 0 & e^{\frac{i\Pi\alpha'\varepsilon}{2}} & e^{\frac{i\Pi\alpha'\varepsilon}{2}} & -e^{\frac{i\Pi\alpha}{2}} & -e^{\frac{-i\Pi\alpha}{2}} \\ 0 & 0 & \mu'\alpha' e^{\frac{-i\Pi\alpha'\varepsilon}{2}} & \mu'\alpha' e^{\frac{i\Pi\alpha'\varepsilon}{2}} & -\mu\alpha e^{\frac{i\Pi\alpha}{2}} & \mu\alpha e^{\frac{-i\Pi\alpha}{2}} \\ 0 & 0 & 0 & 0 & \alpha & -\alpha \end{vmatrix} \qquad (9)$$

Considering the general case for which $a \neq 0$ we may make use of antisymmetry to show that $C_3 = -C_1$, $C_3' = -C_1'$ in which case equation (9) may be reduced to the following 4 x 4 determinant.

$$0 = \begin{vmatrix} 1 & 0 & 0 & 0 \\ e^{\frac{-i\Pi\alpha}{2}} & 2\cos\frac{\Pi\alpha}{2} & -2i\sin\frac{\Pi\alpha'\varepsilon}{2} & -e^{\frac{-i\Pi\alpha'\varepsilon}{2}} \\ \gamma\alpha e^{\frac{-i\Pi\alpha}{2}} & -2i\,\gamma\alpha\sin\frac{\Pi\alpha}{2} & -\alpha' 2\cos\frac{\Pi\alpha'\varepsilon}{2} & \alpha' e^{\frac{-i\Pi\alpha'\varepsilon}{2}} \\ 0 & 0 & 0 & -4i\alpha'\sin\frac{\Pi\alpha'\varepsilon}{2}\cos\frac{\Pi\alpha'\varepsilon}{2} \end{vmatrix} \qquad (10)$$

This may be expanded in the convenient form

$$\gamma\alpha \sin\frac{\Pi\alpha}{2} \sin\frac{\Pi\alpha'\varepsilon}{2} - \alpha' \cos\frac{\Pi\alpha}{2} \cos\frac{\Pi\alpha'\varepsilon}{2} = 0 \tag{11}$$

Following the same procedure, but now considering motion which is symmetric with respect to the core midplane, we obtain the following dispersion equation

$$\gamma\alpha \cos\frac{\Pi\alpha}{2} \cos\frac{\Pi\varepsilon\alpha'}{2} - \alpha' \sin\frac{\Pi\alpha}{2} \sin\frac{\Pi\varepsilon\alpha'}{2} = 0 \tag{12}$$

For the cases where the outer edges of the faces are displacement-free rather than traction-free the following two dispersion equations are obtained.

$$\gamma\alpha \cos\frac{\Pi\alpha}{2} \sin\frac{\Pi\varepsilon\alpha'}{2} + \alpha' \sin\frac{\Pi\alpha}{2} \cos\frac{\Pi\varepsilon\alpha'}{2} = 0 \tag{13}$$

$$\gamma\alpha \sin\frac{\Pi\alpha}{2} \cos\frac{\Pi\varepsilon\alpha'}{2} + \alpha' \cos\frac{\Pi\alpha}{2} \sin\frac{\Pi\varepsilon\alpha'}{2} = 0 \tag{14}$$

where (13) represents the case when the motion is antisymmetric with respect to the core midplane and (14) represents the case when the motion is symmetric with respect to the core midplane.

Inspection of Brillouin Zone Edges

These same four dispersion equations (11-14) can also be obtained by inspection of the edges of the Brillouin zones for the problem involving SH wave propagations in a periodically layered infinite elastic body (Fig. 2). The propagation of SH waves in such a medium has been analyzed in some detail in reference (2). The dispersion relation for the infinite medium represents a surface in frequency -

wave number space and is shown on the extended zone scheme in Fig. 3. As explained in reference (2) the motion at the edges of the Brillouin zones is periodic in the direction perpendicular to the layering. The two upper graphs in Fig. 5 show mode shapes at points of the end of the first brillouin zone.

The two lower graphs in Fig. 5 show mode shapes at points of the end of the second Brillouin zone. It should be noted that wave motion at the edges of the Brillouin zone represents standing waves. The titles in Fig. 5 i.e., asym-sym, sym-asym, etc., indicate the symmetries which exist in the layer with unprimed constants and the layer with primed constants, respectively. For example, looking at the figure in the upper right hand corner, the designation sym-asym indicates that the motion is always symmetric with respect to the layer midplane in the layer with unprimed constants and always antisymmetric with respect to the layer midplane in the layer with primed constants. In view of this fact we may conceptually remove the material, for example, above the midplane of the top layer u and below the midplane of the lower layer u (see Fig. 6a). This leaves a "symmetric sandwich plate" consisting of a core of material with primed constants and thickness $2h'$ and two facings of thickness h made of material with unprimed constants whose outer surfaces are always stress free. The dispersion equation for this symmetric - asymmetric motion is given by equation (21) of reference (2). This equation is identical to equation (11) which was obtained by solving the boundary value proplon earlier in this paper. Equation (19), (20) and (22) of reference (2) represent the dispersion relations for the motions depicted in the other three graphs in

Fig. (5) and are identical to equations (12), (13) and (14), respectively, of this paper. It is interesting to note that equation (11) can also represent the dispersion relation for a different system than the one already mentioned. Using Fig. 6b as an aid, consider a symmetric sandwich plate with facings of thickness h' and a core of thickness 2h. In contrast to before the facings will now be constructed from material with unprimed constants. In addition we will stipulate that the motion is symmetric with respect to the core midplane and the outer surfaces of the facings will now be displacement free instead of stress free. By looking at the lefthand curves of Fig. 6, it is obvious that this system must be described by the same dispersion relation as for the case previously mentioned. Similar observations may be made for the other curves in Fig. 5.

Conclusion

It has been shown that certain parts of the dispersion spectrum governing horizontally polarized shear waves in an infinite, periodically layered medium, can be interpreted as dispersion relations describing motions of symmetric sandwich plates with traction-free or displacement-free outer faces. This finding provides further insight into the dynamic behavior of periodically layered composites.

Acknowledgment

This research was supported in part by ONR contract N00014-76-C-0054.

References

1. Delph, T.J., Herrmann, G. and Kaul, R.K., "Harmonic Wave Propagation in a Periodically Layered, Infinite Elastic Body: Antiplane Strain," Journal of Applied Mechanics, 45 (1978) 343-349.

2. Lee, P.C.Y. and Chang, N., "Harmonic Waves in Elastic Sandwich Plates," Research Report No. 77-SM-14, Department of Civil Engineering, Princeton University.

Captions of Figures

Fig. 1 Geometry of symmetric sandwich plate

Fig. 2 Geometry of layered elastic solid

Fig. 3 Antiplane strain dispersion surface

Fig. 4 Spectral lines corresponding to modes of opposite symmetry, at the ends of the Brillovin zones.

Fig. 5 Mode shapes at points on the ends of Brillouin zones ($\zeta = 0$).

Fig. 6 (a, b) Comparison of mode shapes of a sandwich plate and of an infinite medium for two types of boundary conditions.

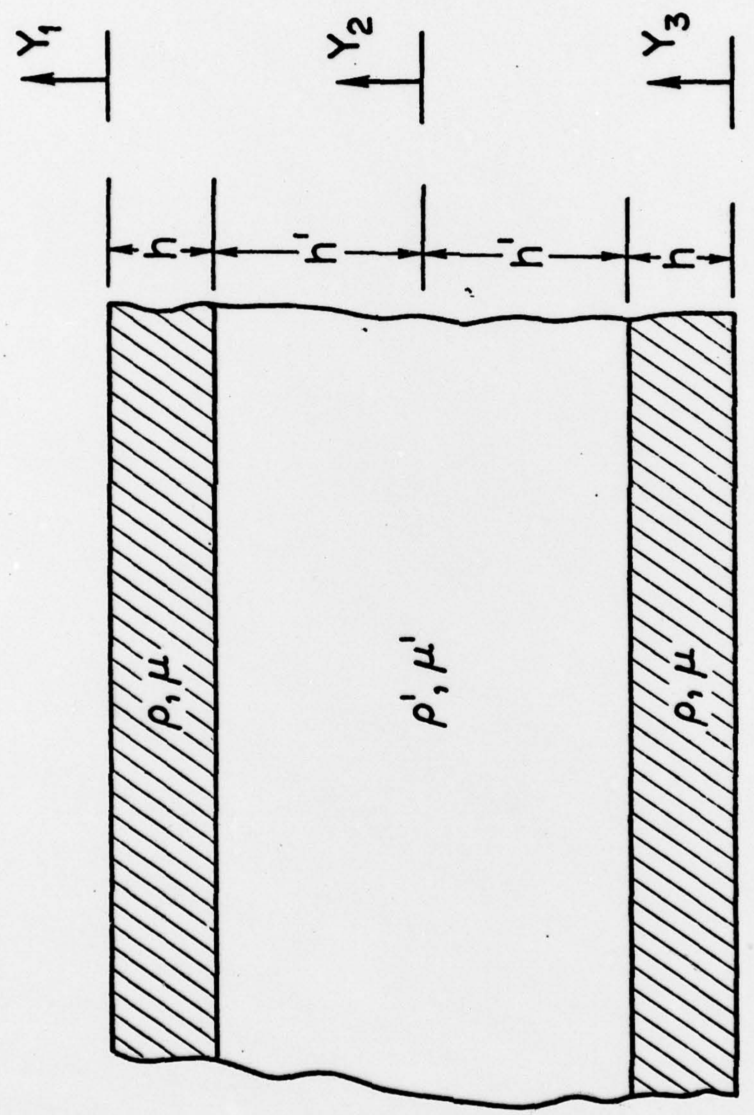


Figure 1

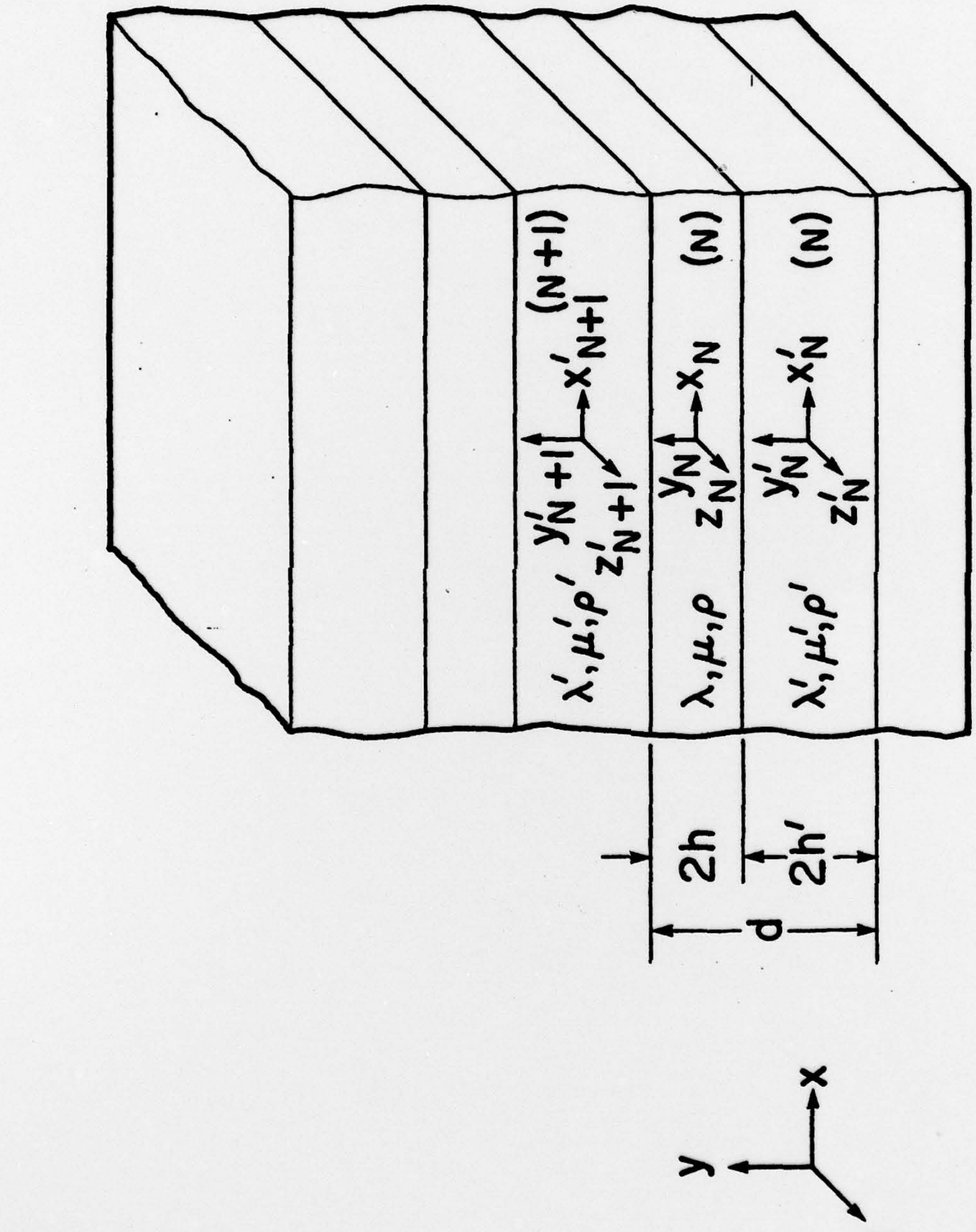


Figure 2

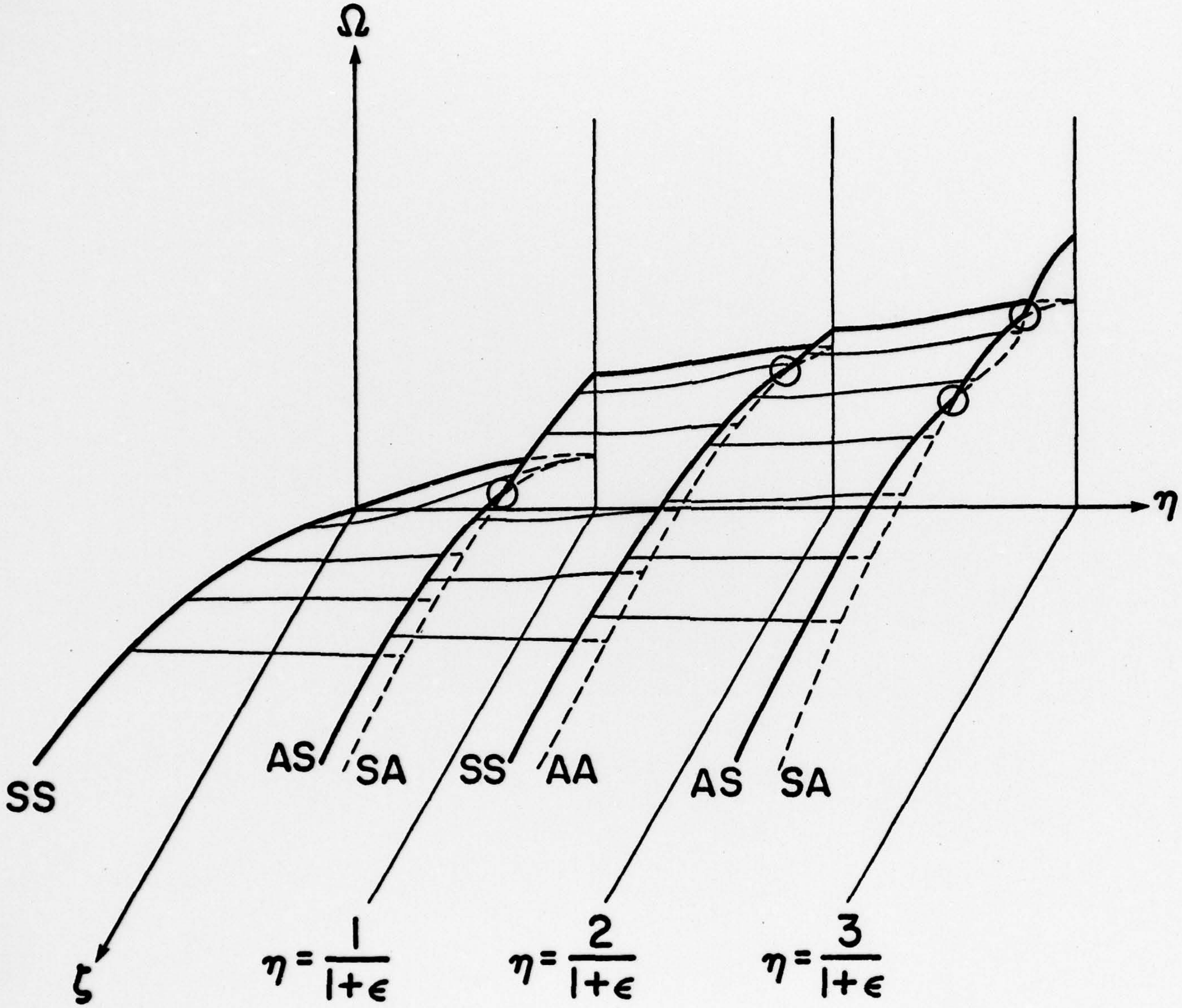
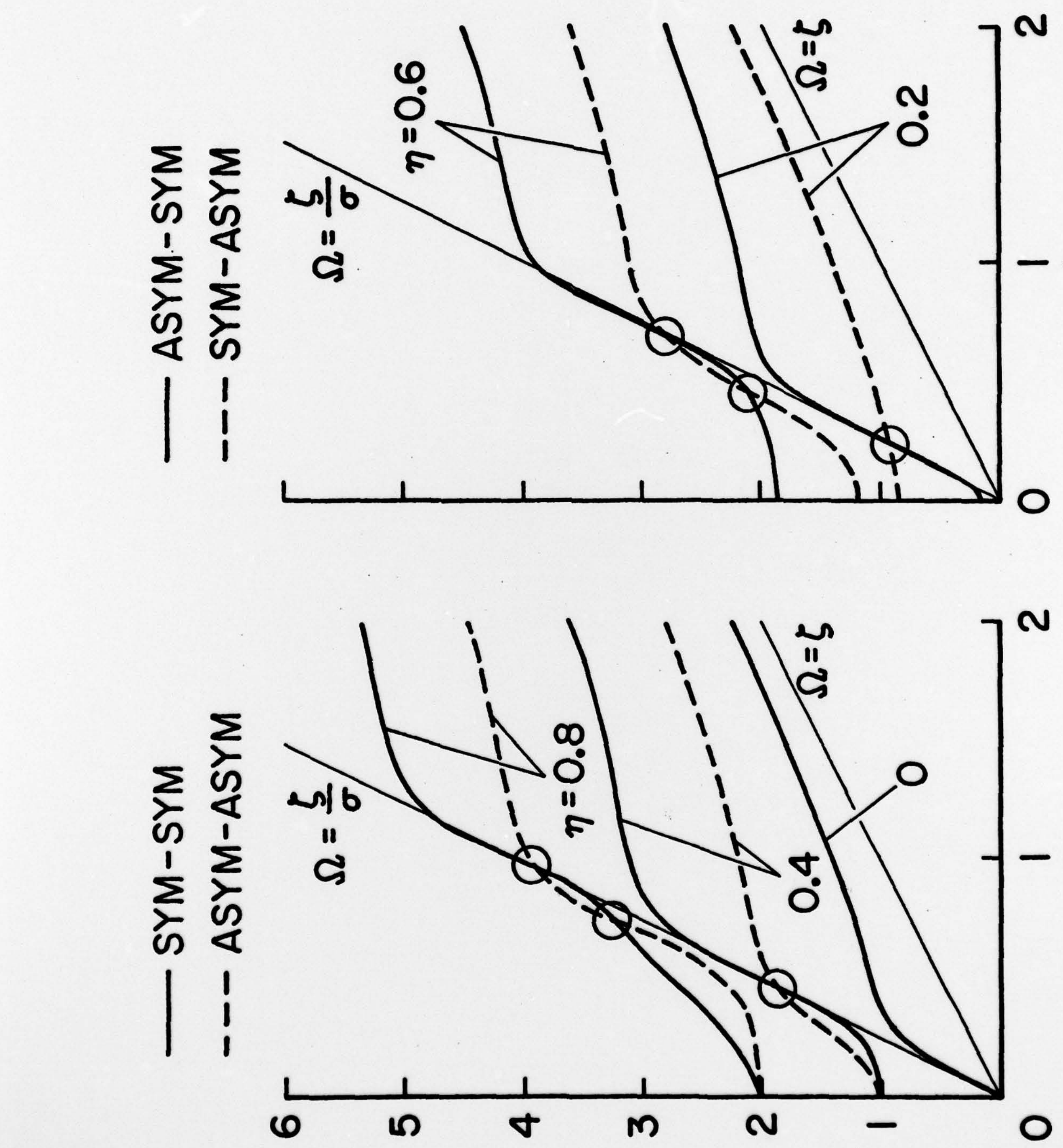


Figure 3



SYM-SYM
ASYM-ASYM
ASYM-SYM
SYM-ASYM
$\Omega = \frac{\zeta}{\sigma}$
$\Omega = \zeta$
$\eta = 0.8$
0.4
0
$\eta = 0.6$
0.2
Ω
ζ
0
1
2
3
4
5
6

Figure 4

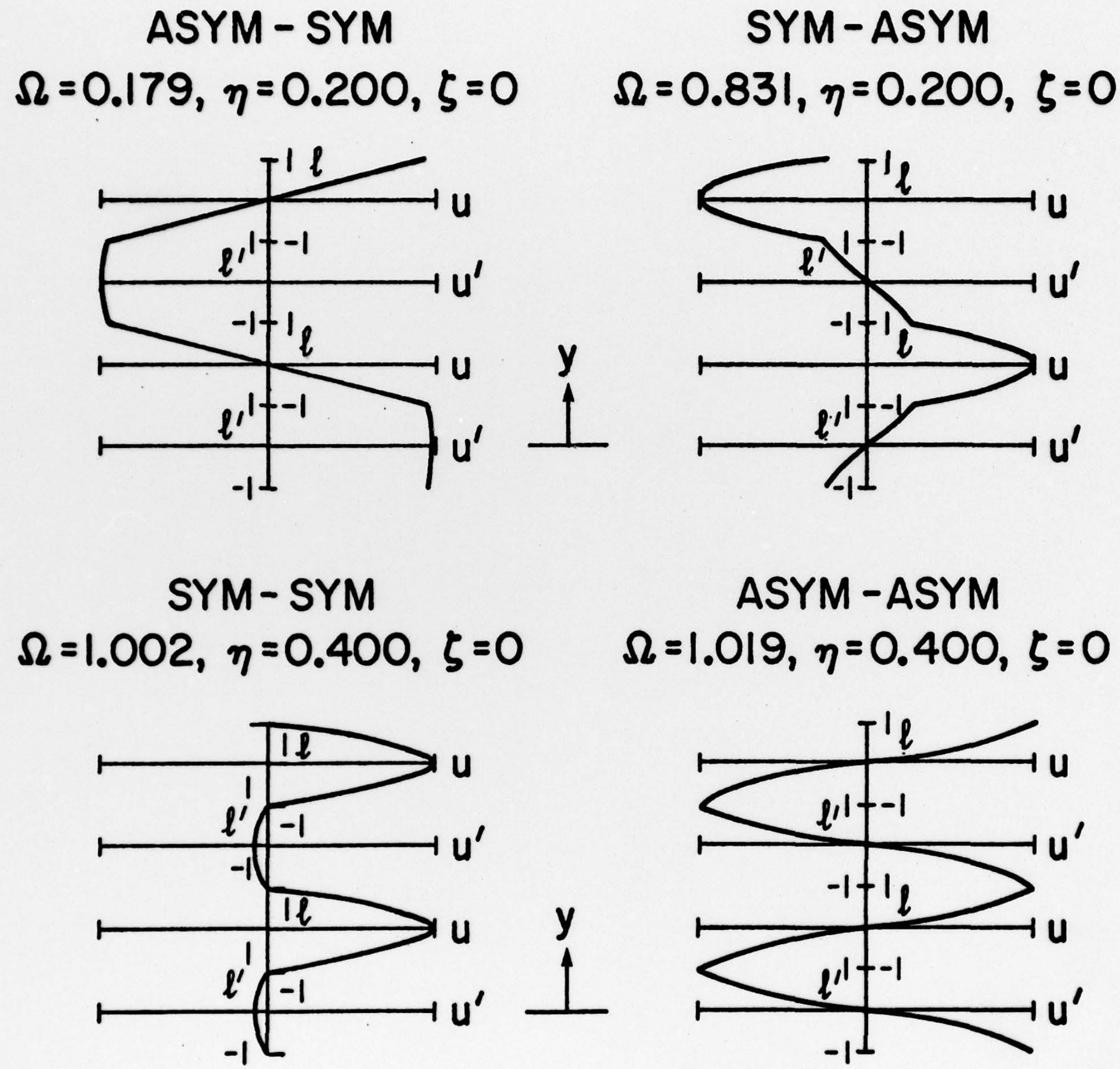

ASYM - SYM
Ω=0.179, η=0.200, ζ=0
SYM - ASYM
Ω=0.831, η=0.200, ζ=0
SYM - SYM
Ω=1.002, η=0.400, ζ=0
ASYM - ASYM
Ω=1.019, η=0.400, ζ=0
u
u'
ℓ
ℓ'
y
1
-1

Figure 5

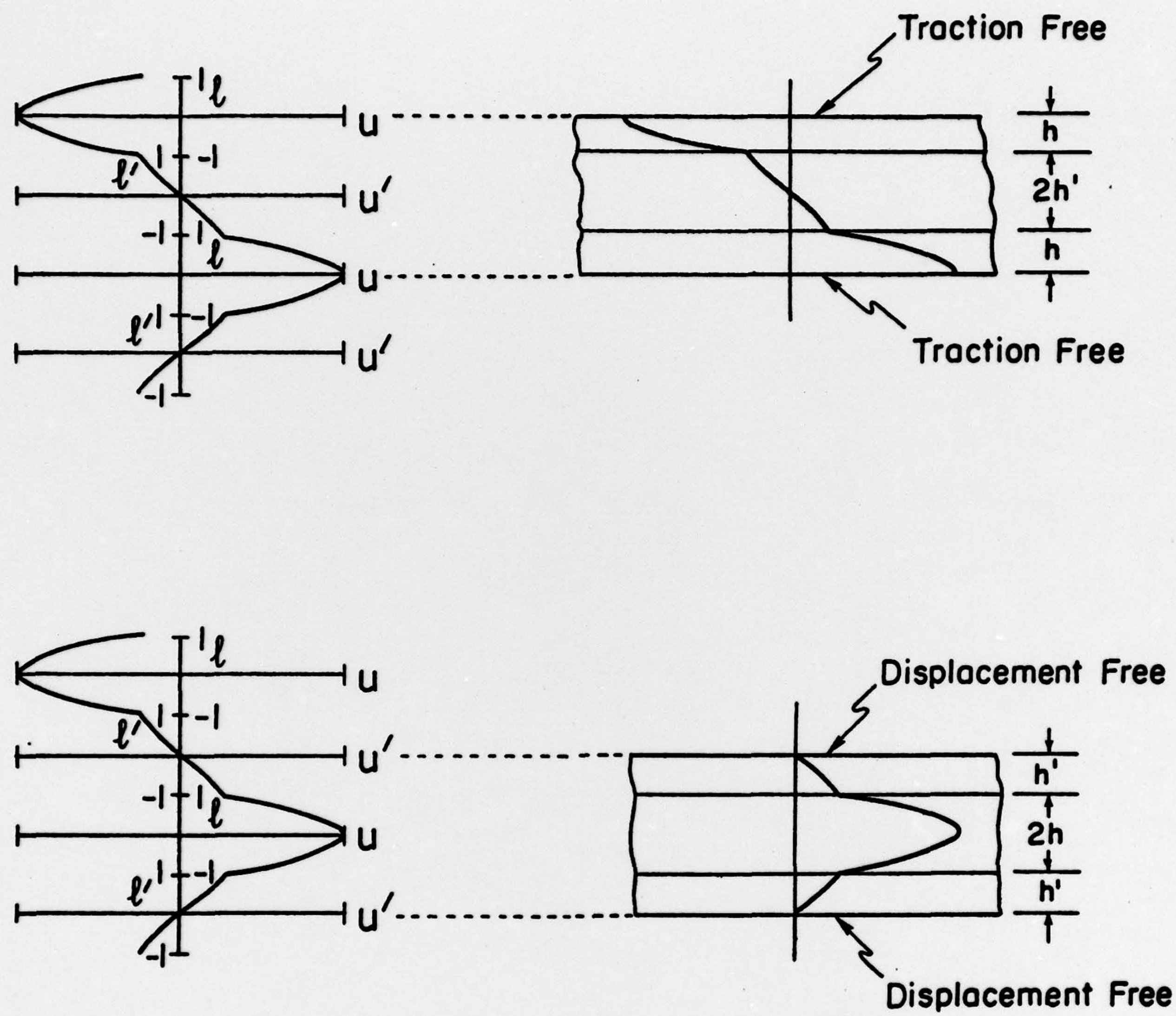

Traction Free
h
2h'
h
Traction Free
u
u'
u
u'
Displacement Free
h'
2h
h'
Displacement Free
u
u'
u
u'

Figure 6